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Vortical structures around a flexible oscillating panel for maximum thrust in a quiescent fluid



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A R T I C L E I N F O

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ABSTRACT

It has been agreed that a proper level of flexibility in moving appendages like a fin of swimming animals enhances their propulsive performance. However, a few efforts have been spent to characterize the criterion, as a simple guideline for designing a biomimetic propulsor, at which the beneficial effect of compliance is maximized. Recently, it was reported that a sinusoidally pitching panel produces the enhanced thrust when the passively bending angle of a trailing edge lags behind its pitching angle by around $\pi/2$ due to its compliance. To understand its mechanism, we perform a series of particle image velocimetry measurements around a panel pitching in quiescent water, while varying its compliance, planform shape and pitching frequency. For all the planform shapes and frequencies considered, with a phase delay of about $\pi/2$, the region of high streamwise velocity with thrust-generating momentum is retained farther (in the streamwise direction) in the wake, caused by the large effective pitching angle during the accelerating stage of pitching rotation. When the panel is stiffer (or the phase delay is smaller than $\pi/2$) than the optimal condition, however, a strong interaction between the trailing-edge vortices (TEVs) formed successively at each half strokes pushes the surrounding fluid into the transverse direction, thereby accelerating the decay of thrust-generating streamwise velocity. This interaction between TEVs is weak for the case of optimal compliance. On the other hand, in the case of over-compliance, the trailing edge of the panel rotates opposite to the pitching direction, which indicates that the inertial work required to rotate the panel during the strokereversal becomes excessive at the expense of rotational circulation which eventually weakens the thrust.

1. Introduction

The subject of understanding the underlying flow physics in propulsion by an oscillatory motion of fins or wings is of both biological and engineering interests (Fish and Lauder, 2006; Shyy et al., 2010). From a biological standpoint, such knowledge has enabled us to investigate fundamental questions about the swimming and flying gaits in nature (Triantafyllou et al., 2000; Kweon and Choi, 2010; Park and Choi, 2012) and energetics of locomotive behaviors (Fish and Lauder, 2006). This, in turn, has enhanced the design, control, and efficiency of man-made small-scale underwater (or air) vehicles (Rozhdestvensky and Ryzhov, 2003).

During the past years, in particular, there have been considerable efforts to understand and mimic the propulsive dynamics of aquatic animals (Epps et al., 2009; Low and Chong, 2010; Esposito et al., 2012; Moored et al., 2012). In the meanwhile, various

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aspects of a rigid oscillating panel (or foil), important to the production and control of thrust, have been investigated such as the vortex dynamics involved in the thrust-generating mechanism (Koochesfahani, 1989; Ahlborn et al., 1997; Dabiri, 2009; Green et al., 2011; Kim and Gharib, 2011; Lee et al., 2013; Mackowski and Williamson, 2015) and the effect of oscillation frequency on the propulsive efficiency (Triantafyllou et al., 1993; Spagnolie et al., 2010; Moored et al., 2012; Lee et al., 2013). When a panel undergoes either a pure heaving or pitching, or a heaving with pitching, it produces a thrust force if its oscillation frequency or amplitude is large enough (Koochesfahani, 1989; Fish and Lauder, 2006; Green et al., 2011). As Dabiri (2009) proposed that the formation and organization of vortex is a unifying principle of the fluid–structure interaction in biological locomotion, the mechanism of thrust generation has been associated with the dynamics of pairs of spanwise vortices in the wake. In a thrust-generating case, these vortices have been shown to be aligned such that the induced velocity is directed downstream (so-called in a reverse von Kármán vortex street), thereby jetting a streamwise momentum into the wake (Koochesfahani, 1989; Triantafyllou et al., 2000; Green et al., 2011).

Among many other issues in biological or bio-inspired propulsors, the hydrodynamic role of structural flexibility has received much attention recently as common characteristics of moving appendages in swimming animals (Lauder, 2011). Elastic force due to the flexibility, coupled with the hydrodynamic and inertial forces creates the deformation of a panel during the oscillating motion which, in turn, affects the hydrodynamic performance. Currently, it is agreed that there exists an optimal compliance for a given oscillation frequency and amplitude; i.e., enhanced thrust generation or corresponding power efficiency is obtained at a certain level of compliance, compared to the rigid counterpart (Heathcote et al., 2004; Heathcote and Gursul, 2007; Spagnolie et al., 2010; Zhang et al., 2010; Kang et al., 2011; Ramananarivo et al., 2011; Ferreira de Sousa and Allen, 2011; Esposito et al., 2012; Park et al., 2012; Dewey et al., 2013; Hua et al., 2013; Bergmann et al., 2014; Cleaver et al., 2014; Moored et al., 2014; Quinn et al., 2014, 2015; Yeh and Alexeev, 2014; Akkala et al., 2015). In the meanwhile, several studies have tried to establish the condition to maximize the beneficial impact of flexibility (e.g., Kang et al., 2011); however, it is still under progress to provide it working under a wide range of environments. This is mostly attributed to the fact that the propulsive performance of a flexible oscillating panel is a function of the oscillation frequency and amplitude, and the passive motion of a trailing edge (i.e., phase difference caused by the interaction between the flexibility and surrounding fluid) whose contributions are interconnected in a very complex manner (Quinn et al., 2015). For example, Zhang et al. (2010) proposed that a phase shift between the heaving and passive pitching angles, and a ratio of natural frequency of the system to the heaving frequency determine the performance of a heaving (with passive pitching of the trailing edge) flat plate. Dewey et al. (2013) showed that the propulsion efficiency of a flexible pitching panel is maximized when its oscillation frequency satisfies two conditions: (i) the dimensionless frequency normalized by the amplitude of the motion of trailing edge and the free-stream velocity should be in the optimal range of 0.25-0.35 (Triantafyllou et al., 1993) and (ii) it should also match the structural resonance frequency of the panel. Moored et al. (2014), applying a linear stability analysis to the time-averaged jet-flows generated by a flexible pitching panel, showed that the optimal flexibility is achieved when the structural resonance frequency is matched to the wake resonance frequency. Extending the work by Dewey et al. (2013), Quinn et al. (2014, 2015) have investigated the propulsion efficiency of a flexible heaving (or heaving and pitching) panel with a free-stream. Due to the relatively long chord length of the panel considered, they were able to produce the bending motions of higher modes, and for a certain condition corresponding to the enhanced thrust generation, the attached flows on the panel were observed, being accompanied with the maximum amplitude of the trailing edge. On the other hand, Hua et al. (2013) performed a numerical simulation on a flexible foil that is forced to heave at the leading edge in a stationary fluid. They showed that the foil moves forward, backward or irregularly depending on the combined effects of the heaving frequency and amplitude, and bending rigidity. Very recently, Feilich and Lauder (2015) also showed that the complicated interactions from the planform shape and compliance are important in determining the propulsive performance of undulating foils.

Considering the complex interplay among the parameters that affect the performance of a compliant oscillating panel, a simple criterion for maximum performance is necessary as a practical guideline for designing an undulating propulsor. For this purpose, a relatively simple, so-called half- π phase delay condition for structural flexibility to enhance the thrust-generation on a harmonically pitching panel in a quiescent water was suggested, as a possible candidate, from a recent experimental study of Park et al. (2012). Observing that the trailing edge of such a compliant panel bends also harmonically, they focused on the phase difference between the pitching and passively bending angles (see Fig. 1a for their definitions). They measured the thrust forces on different conditions and found that the maximum thrust is achieved when the panel's flexibility creates a phase difference of about $\pi/2$ (the bending angle lags behind the pitching angle), when the thrust measurement data are gathered for a wide range of the geometry (planform shape and aspect ratio) and kinematics (pitching frequency). To deepen our understanding, we think it is necessary to further investigate the changes in the flow fields associated with the compliance and identify the mechanism (or flow structures) behind the suggested optimal condition.

In the present study, we identify the flow structures of generating maximum thrust at the condition dictated by half- π phase delay and explain its working mechanism. We perform a series of particle image velocimetry measurements around a pitching panel while varying its flexibility, planform shape and pitching frequency, and analyze the time-averaged and instantaneous flow fields together with the previous force data. The flexible panels and experimental conditions are identical to those used in Park et al. (2012).

2. Experimental setup and procedure

2.1. Pitching kinematics

Fig. 1a illustrates the present pitching kinematics that is driven sinusoidally, centered at the leading edge of a peduncle-like part, by a brushed DC-motor and scotch-yoke mechanism (Park et al., 2012). In this setup, the pitching (ψ) and bending (φ) angles are



Fig. 1. (a) Pitching kinematics with the definitions of pitching (ψ) and bending (φ) angles. (b) Considered planform shapes. In (b), all the shapes have the same planform area and maximum chord length.

fitted as:

$$\psi = A_p \cos(2\pi f t),\tag{1}$$

$$\varphi = A_b \cos(2\pi f t - \xi). \tag{2}$$

Here, A_{ρ} (=15°) and A_{b} are the amplitudes of the pitching and bending angles, and ξ is the phase difference between the pitching and bending angles. Also, for further discussion of the following results, we define the effective pitching angle $\psi_{eff} = \psi + \varphi$, as a measure of actual passive deformation experienced by the trailing edge.

In Park et al. (2012), they varied the pitching frequency (*f*) from 0.5 to 3.2 Hz and showed that the half- π phase delay condition is valid for the range of *f* considered to produce ξ between 0.1π and 0.75π . Based on this finding, in the present study, the case of f = 1 Hz is considered as a reference, which is chosen in order to focus on the instantaneous flow fields around the considered panels with a reasonable time resolution, especially for the case of optimal flexibility ($\xi = 0.5\pi$), with a given framing speed of available CCD camera. In addition, we also test the cases of f = 0.5 Hz and 2.2 Hz to confirm the independency of present results on the pitching frequency that was proposed previously by Park et al. (2012). From the combination of different pitching frequency and the flexibility of the panel (Section 2.2), therefore it is possible to measure the flow fields around the flexible panels whose phase difference covers the range of $0.1\pi < \xi < 0.63\pi$ (see Table 2).

As has been stated in many previous studies (Ahlborn et al., 1997; Lai and Platzer, 2001; Heathcote et al., 2004; Kim and Gharib, 2011; DeVoria and Ringuette, 2012; Lee et al., 2013), the problem of oscillating panel under the zero-free-stream condition has its own interest to be investigated to enhance our existing knowledge in propulsion by oscillation. For example, the present zero-free-stream condition corresponds to the situation when the fish starts from the rest or initiates a fast sprint (so-called a fast-start condition), which has motivated the previous study of Ahlborn et al. (1997). As discussed in detail by Kim and Gharib (2011), the zero-free-stream condition is also extended to understand the flow physics related to the drag-based thrust generation (e.g., pedaling).

2.2. Flexible panels

In the study of Park et al. (2012), the thrust measurement has been performed on four different planforms (with different aspect ratios) while maintaining the same wetted area (Fig. 1b). Here, we use the same planforms as those in Park et al. (2012). Considering that the influence of compliance (i.e., variation of the thrust with phase difference) was not affected by the considered panel geometries, one of them (shape F1, with which we intend to simplify and mimic the caudal fin of a dolphin, representing thunniform swimmers in nature, Sfakiotakis et al., 1999) is considered as a reference case and other three shapes (F2–F4) are additionally tested to check the effect of different shape, especially that of the tip region. Based on this variation, we can also indirectly validate that the velocity measurement at the center-plane can explain the trend of force measurement. If three-dimensional flow structures such as a tip vortex are strong, the flow structure at the center-plane would be different depending on the panel shape and would not match the trend of thrust measurement. For each shape, the span (*S*) is 120 mm, 110.3 mm, 77.1 mm and 134.2 mm, while their planform areas (*A*) are fixed as 3855 mm², respectively, and thus their aspect ratios ($AR = S^2/A$) are 3.7, 3.2, 1.5 and 4.7, respectively. The maximum chord length (*c*) is kept same as 50 mm at the mid-span.

In the present setup, the most important variable is the stiffness of the panel, which denotes the resistance to deformation, as has been defined conventionally. Since the present panel deforms mostly in a chordwise direction, the stiffness denotes the bending stiffness actually. As shown in Table 1, we consider nine different cases (from T1 to T9) whose stiffness is controlled by different combinations of material and thickness. For example, among considered, the model T1 (0.4 mm thick polyvinyl chloride) is the most flexible and T9 (1.7 mm thick acrylic) is the stiffness. Here, the stiffness of each panel is measured using a universal test machine and

Table	1
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Characteristics of the different stiffnesses for panels considered in the present study.

Stiffness variation	Material (thickness)	Stiffness (N/m)
Т1	Polyvinyl chloride (0.4 mm)	102
Т2	Polypropylene (0.5 mm)	171
Т3	Polyvinyl chloride (0.55 mm)	236
Τ4	Polypropylene (0.7 mm)	292
Т5	Polyvinyl chloride (0.75 mm)	775
Тб	Polypropylene (1.0 mm)	803
Т7	Polyvinyl chloride (1.0 mm)	1666
Т8	Polypropylene (1.3 mm)	1950
Т9	Acrylic (1.7 mm)	8780

Table 2

Configurations of the tested panels with their phase difference (ξ) and time-averaged thrust coefficient ($\overline{C_T}$).

Panel	f (Hz)	ξ (rad)	\overline{C}_T	Panel	f (Hz)	ξ (rad)	\overline{C}_T
F1T1	1.0	0.503π	4.92	F3T6	1.0	0.172π	2.72
F1T2	1.0	0.372π	4.65	F3T9	1.0	0.083π	2.03
F1T3	1.0	0.369π	4.61	F4T1	1.0	0.398π	4.43
F1T4	1.0	0.353π	4.38	F4T4	1.0	0.261π	3.52
F1T5	1.0	0.267π	3.37	F4T6	1.0	0.202π	2.44
F1T6	1.0	0.280π	3.35	F4T9	1.0	0.077π	1.79
F1T7	1.0	0.251π	2.82	F1T1	0.5	0.169π	2.77
F1T8	1.0	0.242π	2.63	F1T1	2.2	0.626π	2.83
F1T9	1.0	0.124π	2.29	F1T4	0.5	0.118π	2.60
F2T1	1.0	0.388π	4.49	F1T4	2.2	0.411π	4.01
F2T4	1.0	0.225π	3.34	F1T6	0.5	0.063π	1.73
F2T6	1.0	0.167π	2.55	F1T6	2.2	0.401π	3.98
F2T9	1.0	0.143π	1.97	F1T9	0.5	0.013π	1.59
F3T1	1.0	0.299π	3.78	F1T9	2.2	0.331π	2.39
F3T4	1.0	0.257π	3.32				

the details are found in Park et al. (2012).

Therefore, combining the different shapes (F1–F4), stiffnesses (T1–T9) and the pitching frequencies (0.5, 1.0 and 2.2 Hz), we measure the velocity fields around 29 different cases (see Table 2). In Table 2, the phase difference (ξ) and the time-averaged thrust coefficient (\overline{C}_T) corresponding to each condition are given together, which were measured in Park et al. (2012), to help understanding of the present results. Here, \overline{C}_T is defined as $\overline{C}_T = T/(0.5\rho U_0^2 A)$, where \overline{T} is the time-averaged thrust force, ρ is the density of water, and U_0 is the mean translating velocity of the panel trailing edge for a rigid panel. The thrust force was measured (at 2000 Hz sampling rate) by summing the force data from two force transducers (Ktoyo 333FB), aligned along the *y*-direction (see Fig. 3 and Park et al. (2012) for more details). Since the present experiments are performed in a zero-free-stream condition, it is reasonable to define the characteristic velocity based on the movement of the panel, as has been commonly used in previous studies to investigate the flow structures around undulating fins/wings in the zero-free-stream condition (Ahlborn et al., 1997; Lai and Platzer, 2001; Heathcote et al., 2004; Poelma et al., 2006; Kim and Gharib, 2011; DeVoria and Ringuette, 2012; Park and Choi, 2012; Lee et al., 2013). In Table 2, the name of each panel describes the tested configuration; e.g., F1T1 denotes the planform shape F1 with the stiffness T1. It is noted that the case of planform shape F1 that pitches at the frequency of 1.0 Hz is considered as a reference to study the effect of flexibility (Section 3) and then other cases are supplemented to investigate the effects of planform shape (Section 4) and pitching frequency (Section 5).

Fig. 2 shows the time histories of pitching and bending angles for the selected panels F1T1–F1T9 (at f = 1 Hz) (Park et al., 2012). As expected, the bending angles lag behind the pitching angle and their phase difference decreases as the panel becomes stiffer. For example, the bending angle reaches the maximum peak at $t/T \simeq 0.25$, 0.14, 0.1 and 0.02 for the panels F1T1, F1T4, F1T6 and F1T9, respectively. On the other hand, for an oscillating foil in a zero-free-stream condition, the frequency Reynolds number (Lu and Liao, 2006) is calculated as $Re_f = fc^2/\nu = 1250-5500$ for the present experiments, where ν is the kinematic viscosity of water.

In the present study, we are mainly interested in the effect of passive chordwise bending of the flexible panel during undulation, but very slight bending in spanwise (z) direction (e.g., cupping motion) is observed for the model T1 (most flexible one among considered). Although the spanwise distortion of the panel also affects the vortical structure in the wake (Esposito et al., 2012), we think that it is negligible compared to the effect of chordwise bending in the present cases.

It is finally noted that the real swimmers in nature would not have the exactly same pitching frequency or stiffness of their caudal-fin as those considered in the present study. However, the non-dimensionalized frequency parameter, Re_{f_3} for nature's swimmers that use a caudal-fin oscillation to generate a thrust (carangiform and thunniform) (Sfakiotakis et al., 1999) is calculated to be in the range of 3×10^2 – 2.7×10^5 based on their morphological and kinematic data available in the literature (sunfish, Esposito et al., 2012; mackerel, Gibb et al., 1999; trout, Ahlborn et al., 1991; shark, Graham et al., 1990; tuna, Wardle et al., 1989; and



Fig. 2. Time histories of the pitching (ψ) and bending (φ) angles for the panels F1T1–F1T9 pitching at f = 1 Hz (Park et al., 2012).

dolphin, Fish et al., 2003). Thus, it is confirmed that the present parameters for pitching frequency and panel size belong to those in nature. Nevertheless, the main focus of the present study is to understand the flow structures when the range of considered parameters (stiffness and frequency, in particular) are combined to produce the optimal condition for the maximum thrust, found by Park et al. (2012), not to investigate the hydrodynamics of a specific swimming animal in nature. Feilich and Lauder (2015) also emphasized that it may not be correct to optimize a single parameter such as the panel shape and stiffness but the resultant interaction from them should be targeted. Furthermore, the same optimal kinematic condition, i.e., phase difference of about $\pi/2$ between the pitching and passive bending angles of the trailing edge has been found in nature as well, for the swimming of real cetaceans (Fish et al., 2003). This approach is further supported by Moored et al. (2014) who commented that the swimming animals, regardless of the stiffness of the caudal-fin, may tune its structural resonance to the wake resonance for efficient propulsion.

2.3. Particle image velocimetry

We use a digital particle image velocimetry (DPIV) to measure the velocity fields around the oscillating panels in a transparent acrylic water tank (1 m \times 1 m \times 2 m) (Fig. 3). DPIV is performed by seeding hollow glass spheres (Potters Industries, Inc.) whose mean diameter is about 10 μ m. A delay generator (OPS-SYNC-104P, Optical Systems) controls the synchronization of the image captures by a CCD camera (IPX-4M15-L, Imperx Inc.) and the firing of double-pulsed Nd-Yag laser (Solo PIV, New Wave Research Inc.). The CCD camera has a resolution of 2048×1024 pixels with a double frame capturing mode at 15 Hz. Each pair of particle



Fig. 3. Experimental setup for water tank equipped with a pitching panel and particle image velocimetry equipments.

images, separated by 0.5, 2.0 and 3.0 msec, for f = 2.2, 1.0 and 0.5 Hz, respectively, are evaluated by cross correlation using fast-Fourier-transform algorithm with an interrogation window (64×64 pixels, 50% overlap). When outliers whose size is larger than three times the standard deviation of the mean vector length are detected, they are replaced by vectors that are interpolated from the surrounding vectors (in 3×3 grids). Thus, the field of view has a range of $-0.2 \le x/c \le 3.0$ and $-3.0 \le y/c \le 3.0$ with a spatial resolution of 0.052*c*. Temporal resolution of the current measurement is about 0.125*T* for f = 1 Hz, where *T* is the one-cycle period of the pitching motion (i.e., 1 s). The velocity fields are measured at the mid-span (z/c = 0). Since the aspect ratio of the considered panel is not high, the resulting flow would be three-dimensional, e.g., existence of tip vortices; however, due to the zero-free-stream environment, its impact on the thrust generation is assumed to be not significant compared to that from the trailing-edge vortex measured at the mid-span (DeVoria and Ringuette, 2012; Lee et al., 2013). If the present panels undergo substantial deformation along the spanwise direction, on the other hand, the flow structures will be affected much as well, like in the study of Jain et al. (2015). As explained above, major deformation occurs along the chordwise direction in the present study, and this will not impose a significant influence on the spanwise variation of the flow. Similarly, there are other studies that measured the flow at the centerplane to explain the hydrodynamics of three-dimensional panel/wing oscillating in a quiescent fluid (Alben et al., 2012; Lee et al., 2013; Heathcote et al., 2004; Tam, 2015). In fact, in a zero-free-stream condition, the relationship between the thrust and phase difference was not affected much by the planform shape (Park et al., 2012), and we will discuss this later (see Section 3.1).

3. Effects of flexibility: mechanism of half- π delay condition

3.1. Flow fields at different spanwise locations

As noted in previous sections, we will focus on the velocity fields measured at the center-plane, i.e., z/c = 0, to understand the combined effects from various parameters on the flow structures that are dominantly responsible for the thrust generation. Since the present flexible panels have a relatively low aspect ratio, in order to validate our approach, it is first necessary to look into the spanwise variation of the flow fields. Thus, for the panel F1T1 pitching at the frequency of 0.5, 1.0 and 2.2 Hz, the velocity fields are measured at five positions along the spanwise direction, z/c = 0 (center-plane), 0.2, 0.5, 0.8, and 1.2 (tip). Here, we plot the contours of time-averaged (obtained by averaging about 500 instantaneous velocity fields) streamwise velocity for pitching at 0.5 Hz (Fig. 4a) and time-averaged spanwise vorticity for 1.0 Hz (Fig. 4b) and 2.2 Hz (Fig. 4c). After tracking the centerline velocity (i.e., u at y/c = 0), we find that it becomes approximately periodic after the third stroke. Thus, the data after the third stroke per each trial are used to get the time-averaged flow statistics to avoid the inclusion of data during the transient state and ensure the convergence of flow statistics. For the range of considered frequency, it is measured that the same flow structures are retained for a wide spanwise region (about 70% of the span) except very near the tip (z/c = 1.2). That is, the region of thrust-generating high streamwise velocity is concentrated in the wake while the slightly negative streamwise velocity is induced at the sides of pitching panel (Fig. 4a). Also, the coherent flow structure of a pair of counter-rotating spanwise vortex is commonly observed for the region of $0 \le z/c \le 0.8$ (Figs. 4b) and c). On the other hand, the magnitude of streamwise velocity and spanwise vorticity at z/c = 1.2 is very small compared to those at other locations, while the flow structures at the center-plane are most dominant. This is mostly due to the fact that the present flexible panel moves under a zero-free-stream condition and the similar discussion has been given by DeVoria and Ringuette (2012) and Lee et al. (2013). Furthermore, in the detailed velocity measurements by Green et al. (2011), it was shown that almost twodimensional vortex filaments are alternatively generated behind a pitching trapezoidal panel, even with a moderate free-stream velocity. That is, except the tip position, the same flow structures are generated along the spanwise direction. Indeed the absolute values of the flow statistics such as the streamwise velocity and spanwise vorticity vary along the spanwise direction; however, based on the above understandings we think the flow structures measured at the center-plane will provide us a meaningful background to study the influences of flexibility on the thrust generation.

3.2. Time-averaged flow fields

In this section, we discuss the time-averaged flow fields (obtained by averaging about 500 instantaneous velocity fields) in the wake behind pitching panels F1T1–F1T9 pitching at f = 1 Hz. As explained above, the planform shape F1 and pitching frequency of 1 Hz are analyzed as a reference condition to see the effect of stiffness (T1–T9). Contours of the mean streamwise velocity (\overline{u}/U_0) for selected panels are shown in Fig. 5. As shown, the region of high streamwise velocity (i.e., being responsible for the thrust generation) in the near wake is widest for the panel F1T1 ($\xi = 0.503\pi$) but it is reduced significantly as the phase difference decreases (i.e., as the panel becomes stiffer). In the case of panel F1T9 ($\xi = 0.124\pi$), the size of this region is reduced significantly, resulting in a small thrust. In addition, the region of small or negative streamwise velocity (denoted by dashed circles for the panel F1T9 in Fig. 5) gets closer to the panel as the phase difference decreases from $\xi = 0.503\pi$ (panel F1T1) to $\xi = 0.124\pi$ (panel F1T9). Overall, the variation of the time-averaged flow field (also see Fig. S1 in the Supplementary Material for the variation of the mean streamwise velocity profiles along the streamwise direction) agrees well with previous thrust measurements (Table 2), supporting the half- π phase delay condition.

3.3. Flow structures associated with thrust generation

Before we discuss the condition of maximum thrust generation, it is necessary to identify the dominant flow structures that are responsible for the thrust generation in the present configuration. Fig. 6 shows the temporal variation in the vortical structures and

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Fig. 4. Time-averaged flow field measured at five spanwise locations of z/c = 0 (top row), 0.2, 0.5, 0.8 and 1.2 (bottom row) during the pitching rotation of panel F1T1: (a) streamwise velocity (\overline{u}/U_0) at f = 0.5 Hz; (b) spanwise vorticity ($\overline{\omega}_z c/U_0$) at f = 1.0 Hz; and (c) spanwise vorticity ($\overline{\omega}_z c/U_0$) at f = 2.2 Hz.

velocity-vector fields for the panel F1T9, the stiffest among the considered, pitching at f = 1 Hz. As the panel rotates, the surrounding fluid is entrained and dragged to the direction (approximately) perpendicular to the panel surface (denoted as a dashed open arrow in Fig. 6b). Then, a shear layer is formed along the panel surface and is gradually shed at the trailing edge into the downstream, forming elongated vortices (Fig. 6c). When the panel decelerates toward the end of half-stroke (Fig. 6d), another vortex whose vorticity has an opposite sign to that of the vortices in Fig. 6b is created on the surface and is then shed as a trailing edge vortex (TEV) when the rotation is reversed (Fig. 6e). Through the interaction between these TEVs, the fluid entrained during previous half-stroke (Figs. 6a–d) is pushed down to the streamwise direction to impart substantial momentum in the wake, thereby inducing a jet-like flow structure (denoted as a solid open arrow in Figs. 6e and f) which generates a thrust. One thing to note here is that a transverse component of the flow is also induced while pushing the fluid into the streamwise direction. As is discussed below, how the TEVs interact each other and the flow is induced directly depend on the flexibility (i.e., the phase difference and effective pitching angle) of the panel.

3.4. Instantaneous flow fields

Now, we investigate the temporal variation of the flow structures around the panels F1T1, F1T4 and F1T9, together with the time history of the thrust coefficient (C_T) (Fig. 7a) measured in Park et al. (2012), during the first- (Fig. 8) and the second-half (Fig. 10)



Fig. 5. Contours of the mean streamwise velocity (\bar{u}/U_0) at z/c = 0 for selected panels (F1T1, F1T4, F1T6 and F1T9) pitching at f = 1 Hz. The gray-colored area denotes the region swept by each pitching panel and the dotted lines represents the deformed shape of each model at zero pitching angle.

stages in one period of pitching rotation. In Fig. 7b, the temporal change in the effective pitching angle is also plotted for the corresponding cases. The pitching frequency is 1 Hz and the panel changes its direction of pitching rotation from counterclockwise to clockwise one at t/T = 0. As shown in Fig. 7, for each panel, the unsteady behavior of C_T is summarized as (i) C_T becomes maximum when the effective pitching angle experienced by the trailing edge is maximum (Fig. 7b); (ii) after the maximum peak, C_T experiences a steep decline; and (iii) all panels except F1T1 produce non-negligible negative thrust forces before the stroke reversal. It is noted that the asymmetry in the force data between the first and second half strokes are caused by intrinsic uncertainties in the scotch-yoke system and deviations in panel geometry (Park et al., 2012). However, the global trend of thrust variation and its time-averaged value would be explained by the present flow-field measurement. As shown in Fig. 7b, compared to the active pitching angle (ψ ; i.e., when the panel undergoes no deformation), it is shown that the effective pitching angle becomes larger as the phase difference increases toward the optimal condition. In particular, the maximum ψ_{eff} appears during the acceleration stage of the pitching rotation, while it gets closer to the instant of maximum angular velocity with optimal condition.

At the acceleration stage (for example, at $t/T \simeq 0.125$; Fig. 8a), the TEVs of the panels F1T4 and F1T9 have already displaced away from the panel surface and interact with the vortices from the previous half stroke; however, the TEV from the panel F1T1 is still attached to the panel surface. As has been found out previously, the attached flow (Quinn et al., 2014) and strengthening of the TEVs (Cleaver et al., 2014) have been attributed to a stronger time-averaged jet flow (i.e., net thrust force). Hence, to examine the effect of TEVs quantitatively, its circulation is calculated for each instantaneous flow field during the first half. The TEV circulation (Γ_{TEV}) is calculated by integrating the spanwise vorticity over all contiguous grid points within a square search area. The search area whose size is 20×20 grid points centered on the TEV center (position of the maximum vorticity) was searched from the trailing edge of the panel to the lower corners of each flow field. The similar approach to determine the contribution of vortex circulation was used in Muijres et al. (2008) and the present results are shown in Fig. 9. Although the temporal resolution of the present PIV measurements is not fine enough to determine the exact timing, for all cases, the Γ_{TEV} becomes the maximum around $t/T \simeq 0.25$, when the panel's angular velocity is the maximum, and then decreases. At $t/T \simeq 0.125$, the Γ_{TEV} of the panel F1T9 is smaller than those of F1T1 and F1T4 whereas the Γ_{TEV} of the panel F1T1 is smaller than others at other instants. A similar reduction of vortex strength with increasing compliance was also reported on a flexible wing model (Zhao et al., 2011). They showed that the magnitude of the leading-edge vorticity decreases with increasing the flexibility of the wing model. As can be imagined, the dynamics of TEV is determined by the movement of the trailing edge (i.e., bending angle) relative to the pitching direction (Figs. 7b and 8a). That is, the panel F1T1 has a smaller effective pitching angle at an early stage ($t/T \leq 0.1$) (Fig. 7b), and thus the appearance and development of TEV is delayed (Fig. 8a). For panels F1T4 and F1T9, however, the effective pitching angle has reached the maximum earlier (larger



Fig. 6. (a)–(f) Temporal variation (time interval of 0.125T) in the instantaneous spanwise vorticity ($\omega_z c/U_0$) contours and velocity vectors around the panel F1T9 pitching at f = 1 Hz, measured at the mid-span.

than panel F1T1, as well) (Fig. 7b), and thus the shed TEV rolls-up quickly (Fig. 8a). The TEV roll-up of panel F1T1 happens later (Fig. 8b) as the effective pitching angle approaches the maximum at $t/T \simeq 0.2$ (Fig. 7b). Similar analysis has been provided by Dai



Fig. 7. (a) Time history of the thrust coefficient (C_T) for the panels F1T1 (--), F1T4 (- - -), and F1T9 (....) pitching at f = 1 Hz (Park et al., 2012). Dashed vertical lines denote the instants when the flow structures are investigated in Figs. 8 and 10. (b) Variations of effective pitching angle (ψ_{eff}) for the panels F1T1 (•), F1T4 ($\mathbf{\nabla}$), and F1T9 ($\mathbf{\nabla}$) pitching at f = 1 Hz. In (b), \star denotes the imposed (active) pitching angle (ψ).

et al. (2012) who numerically investigated the aerodynamic role of passive deformation of an insect wing in a normal hovering motion (i.e., no free-stream and the wing stroke is done in a horizontal plane). They have shown that a smaller chordwise wing deformation (effective pitching angle) is attributed to the delayed growth of the TEV and the decrease of the lift force generation.

On the other hand, C_T shows a different trend from that of the Γ_{TEV} at $t/T \simeq 0.25$ and 0.375. Thus, a different mechanism from that related to the TEV is necessary to explain the variation of C_T at these instants. Figs. 8b and c show the flow structures near and after the instant $(t/T \simeq 0.25)$ of maximum angular velocity. Although the Γ_{TEV} of the panel F1T9 is largest at these instants (Fig. 9), C_T of the panel F1T9 is much smaller than those of F1T1 and F1T4 (Fig. 7a). Since the amount of surrounding fluid entrained by the pitching motion depends on the angular velocity and orientation of the panel, the flow structures at $t/T \simeq 0.25$ would have a dominant influence compared to other instants. That is, the fluid entrained by the panel F1T9 is dragged almost horizontally (denoted by dashed open arrow) due to the small effective pitching angle and is pushed into the horizontal direction mostly (denoted by a solid open arrow) by the interaction between TEVs (Fig. 8b). It is noted that the arrows in Fig. 8 are drawn to assist the qualitative recognition of the flow orientation, and a direct quantitative comparison is not supported. As the phase difference increases to $\pi/2$, on the other hand, surrounding fluid is dragged more into the streamwise direction. This is due to weaker interaction between the TEVs which is evidenced by the smaller Γ_{TEV} at $t/T \simeq 0.25$ (Fig. 9) and longer distance between the TEVs (Fig. 8b) for the panel F1T1 than the other cases.

To examine the effect of flexibility on the interaction between TEVs in the wake in detail, the momentum flux per unit length through the horizontal (M_x) and vertical (M_y) plane are calculated from the instantaneous velocity fields as

$$M_x = \rho \int u^2 \mathrm{d}y,\tag{3}$$



Fig. 8. Instantaneous spanwise vorticity ($\omega_z c/U_0$) contours and velocity vectors at the mid-span of the panels F1T1 (left), F1T4 (middle) and F1T9 (right) at (a) $t/T \simeq 0.125$, (b) 0.25 and (c) 0.375. Vorticity contour level is same as that in Fig. 6.

$$M_{\rm y} = \rho \int v^2 \mathrm{d}x,\tag{4}$$

where u and v are instantaneous velocities in the streamwise and transverse directions, respectively (Heathcote et al., 2004). And the corresponding momentum flux coefficients are defined as



Fig. 9. Temporal variation of the normalized TEV circulation (T_{TEV}/cU_0) for the panels F1T1 (•), F1T4 ($\mathbf{\nabla}$), and F1T9 ($\mathbf{\nabla}$) pitching at f = 1 Hz.

$$C_{Mx} = M_x (0.5\rho U_0^2 l_y)^{-1},$$
(5)

$$C_{My} = M_y (0.5\rho U_0^2 l_x)^{-1}.$$
(6)

Here, l_y and l_x are the lengths in transverse (y) and streamwise (x) directions over which M_x and M_y are actually calculated, respectively. With this normalization, it is possible to compare the amount of entrained flow independent on the size of integration domain.

Based on this, the portion of the fluid entrained in both directions is examined by calculating C_{My}/C_{Mx} , where C_{Mx} and C_{My} are evaluated for $-3.0 \le y/c \le 3.0$ at x/c = 2.5 and $1.97 \le x/c \le 2.91$ at y/c = 1.03, respectively. These positions were chosen because they coincide with the location of trailing-edge vortex (TEV) interaction as we have observed in instantaneous flow fields (Fig. 8), and thus it is considered to provide a clearer view on the effect of trailing-edge vortex on the flow entrainment. As shown in Fig. 10, in general, the ratio is larger than 1.0, which is reasonable considering the zero-free-stream condition. As the flexibility approaches to the optimal condition (panel F1T1), however, less fluid is wasted into the transverse direction and more is induced to generate the thrust force. In addition, this agrees with our explanation such that in optimal condition, the strength of the shed TEV is weak (Fig. 9).

The flow structures around the panel as it decelerates toward the end of half stroke are shown in Fig. 11. In the deceleration stage, there appears a vortex structure (denoted by the arrows in Fig. 11a) which rotates opposite to the TEV. This vortex also interacts with the TEV, which is different from the TEV interaction responsible for the thrust generation explained in the acceleration stage. As shown in Figs. 11a and b, the distance between those two vortices becomes longer as the panel becomes stiffer due to the smaller effective pitching angle (Fig. 7b), indicating a weaker interaction. Thus, for the stiffer panels (e.g., F1T4 and F1T9), the induced flow seems not to generate a meaningful thrust force as denoted by the dashed open arrow in Figs. 11a and b. Although the influence of this induced flow is much smaller than the TEV interaction on the thrust generation, for F1T4 and F1T9, it



Fig. 10. Temporal variation of C_{My}/C_{Mx} for the panels F1T1 (•), F1T4 (∇), and F1T9 (∇) pitching at f = 1 Hz. Here, C_{Mx} and C_{My} are from Figs. 11c and d, respectively.



Fig. 11. Instantaneous spanwise vorticity ($\omega_c c/U_0$) contours and velocity vectors at the mid-span of the panels F1T1 (left), F1T4 (middle) and F1T9 (right) at (a) $t/T \simeq 0.75$, (b) 0.875 and (c) 1.0. Vorticity contour level is same as that in Fig. 6.

seems for the entrained flow to have a detrimental effect on the thrust (Fig. 7a). This trend also coincides with the distribution of small or negative streamwise velocity around the panels (Fig. 4).



Fig. 12. (a)–(f) Temporal variation (time interval of 0.125T) in the instantaneous spanwise vorticity ($\omega_z c/U_0$) contours and velocity vectors around the panel F3T1 pitching at f = 1 Hz, measured at the mid-span.

4. Effect of planform shape

So far, based on the reference cases of planform shape F1 and pitching frequency of 1 Hz, the effects of flexibility on the wake



Fig. 13. Contours of the mean streamwise velocity (\bar{u}/U_0) at z/c = 0 for the panel F1T1 pitching at (a) f = 0.5 Hz; (b) 1.0 Hz; and (c) 2.2 Hz. Phase difference for each case is noted in the figure.

structure have been discussed and the flow structures responsible for the maximum thrust in zero-free-stream condition have been identified. In this section, while fixing f = 1 Hz, we consider additional three planform shapes (F2–F4) for the panel (Fig. 1b) while varying its stiffness as T1, T4, T6 and T9 (Table 1). Among them, the temporal variation in the vortical structures and velocity-vector fields for the panel F3T1, whose planform shape differs mostly from that of the reference shape (F1) such that it has a blunt tip and the smallest aspect ratio, is shown in Fig. 12. Despite the differences in the planform shape, the same flow structures responsible for the thrust generation for shape F1 (Fig. 6) are also observed in the center-plane. For example, the surrounding fluid is entrained and dragged to the direction perpendicular to the panel surface (denoted as a dashed open arrow in Fig. 12a). A long shear layer is formed and shed at the trailing edge, resulting in elongated vortices (Fig. 12c). When the panel decelerates toward the end of halfstroke (Fig. 12d), another vortex whose vorticity has an opposite sign to that of the vortex in Fig. 12a is created weakly on the surface and then sheds as a TEV when the rotation is reversed (Fig. 12e). Through the interaction between those TEVs, the fluid entrained during previous half-stroke (Figs. 12a-d) is pushed down to the streamwise direction to induce a jet flow (denoted as a solid open arrow in Figs. 12a and e), generating a net thrust. This, as discussed in Section 3.1, again supports the current approach that under the zero-free-stream condition, the flow field measured at the mid-span is enough to achieve the goal of present study. For all the additional planforms, the variations in mean streamwise velocity profiles are analyzed further (see Fig. S2 in the Supplementary Material). The absolute values of the flow statistics are different depending on the planform shape, however, it is clearly shown that the velocity profiles have a same dependency on the flexibility as that of a reference case (Fig. 5). Thus, it can be said that once the planform of the panel is selected, the same half- π phase delay condition for the enhanced thrust and its explanation based on the qualitative and quantitative description of the measured flow fields are valid, agreeing with the previous thrust force measurement (Park et al., 2012).

5. Effect of pitching frequency

Next, the effect of pitching frequency (f) on the half- π phase delay condition is discussed. While fixing the planform shape as F1, we vary f as 0.5 and 2.2 Hz, in addition to 1 Hz that has been investigated as a reference case. For each frequency, the stiffness of the panel is also varied as T1, T4, T6 and T9 (Table 1). Phase difference and thrust coefficient corresponding to each case are summarized in Table 2.

Fig. 13 shows the variation of mean streamwise velocity (\overline{u}/U_0) contour for the panel F1T1 with f, which produces optimal flexibility (i.e., $\xi \simeq 0.5\pi$; Fig. 13b), under- ($\xi < 0.5\pi$; Fig. 13a) and over- ($\xi > 0.5\pi$; Fig. 13c) flexibility conditions. As shown, for a given geometry and stiffness, the kinematic condition of $\xi \simeq 0.5\pi$ has the widest region of high streamwise velocity (i.e., contributing



Fig. 14. (a) Variations of effective pitching angle (ψ_{eff}) for the panel F1T1 with f = 0.5 Hz (\Box), 1.0 Hz (•), and 2.2 Hz (Δ). (b) Schematic diagram for bending of the pitching panel with over-compliance ($\xi > \pi/2$) during stroke reversal. In (a), \star denotes the imposed (active) pitching angle (ψ).

to thrust generation) (Fig. 13b). When the pitching frequency is lower and thus the phase difference is smaller than ~0.5 π , the amplitude of jet velocity is reduced substantially (Fig. 13a), agreeing with the thrust force measurement (see Table 2). As the phase difference becomes larger than 0.5 π (i.e., over-compliance case) with increasing *f*, it is found that the thrust-generating streamwise velocity is weakened much, as well (Fig. 13c). For other stiffnesses that have monotonic variation of ξ with *f* (Table 2), it is also found that the region of high streamwise velocity in the wake becomes wider as ξ increases toward 0.5 π (not shown here; see Fig. 16 for additional information). Therefore, as we have discussed at the end of Section 2.2, the pitching frequency alone is not sufficient to optimize the propulsion performance of a flexible pitching panel and it is rather important to characterize how the range of parameters (stiffness and frequency, for example) are coupled to produce the optimal kinematic condition. The difficulty in optimizing the propulsive performance with a single variable has been also suggested by Feilich and Lauder (2015).

In the above discussion, it is interesting to see that the case of $\xi > \pi/2$ (over-compliance) degrades the thrust-generating performance of the flexible pitching panel. Similarly, Hua et al. (2013) showed that a compliant heaving foil in a stationary fluid moves backward (generating negative thrust) or irregularly when it is highly flexible. While the large amplitude of trailing edge has been attributed to the enhanced thrust generation (Bergmann et al., 2014; Cleaver et al., 2014; Yeh and Alexeev, 2014), it was also commented that the beneficial effect from the large displacement of trailing edge is limited to a certain point when the excessive deformation adds a detrimental effect on the force generation (Yeh and Alexeev, 2014). Thus, it would be interesting to investigate the reason why the over-compliance reduces the thrust force. Fig. 14a shows the variation of effective pitching angle for the panel F1T1 with $f = 0.5(\xi = 0.169\pi)$, $1.0(\xi = 0.503\pi)$, and 2.2 Hz ($\xi = 0.626\pi$). At f = 2.2 Hz, the amplitude of the bending angle (~32°) is larger than those at 0.5 Hz (~9°) and 1.0 Hz (~27°). Due to the large phase difference, however, the effective pitching angle is rather reduced to be comparable to that of f = 0.5 Hz. Furthermore, it has a wide plateau of a large value even in the decelerating stage, and during the stroke reversal ($t/T \simeq 0(1.0)$ and 0.5), the effective pitching angle has an opposite sign to the active pitching angle (ψ). This indicates that the trailing-edge of the panel rotates opposite to the pitching direction during the stroke reversal (Fig. 14b).

Fig. 15 shows the temporal variation of instantaneous vorticity contour and velocity vectors for the case of f = 2.2 Hz ($\xi = 0.626\pi$) with the panel F1T1. Based on our discussion, it is understood that the main idea of half- π phase delay condition is to have a larger effective pitching angle during the accelerating stage, close to the instant of the maximum angular velocity, thereby to maintain the TEV attached and to lose less entrained fluid into the transverse direction. Figs. 15a and d show the instants when the panel rotates at the maximum angular velocity. Since ψ_{eff} has a broad peak across $t/T \simeq 0.25$, compared to the optimal or under-compliance cases (Fig. 8b), it is found that the shed TEV is separated farther from the panel and the interaction between TEV from previous half-stroke mostly induce the flow into the transverse direction (marked as an open arrow in Figs. 15a and d). When the direction of pitching is reversed (i.e., between Figs. 15b and c), on the other hand, the trailing-edge of the panel rotates opposite to the pitching direction (Fig. 14), and a coherent vortex is already created even before the panel changes the rotating direction, which induces a strong fluid velocity that is opposite to the direction of next stroke (noted with an arrow in Fig. 15b). In the case of optimal or undercompliance cases (Fig. 11), the formation of this vortex was not captured clearly. Thus the over-compliance increases the inertial



Fig. 15. (a)–(f) Temporal variation (time interval of about 0.17*T*) in the instantaneous spanwise vorticity ($\omega_z c/U_0$) contours and velocity vectors around the panel F1T1 pitching at f = 2.2 Hz, measured at the mid-span.

work required to decelerate and re-accelerate the panel at the expense of rotational circulation which will significantly weaken the TEVs and then much less fluid is induced and dragged for the thrust generation This is also supported by the comment from



Fig. 16. (a) Variation of \overline{C}_{Mx} (calculated at x/c = 2.5) with phase difference (ξ). (b) Relation between \overline{C}_{Mx} and \overline{C}_T . \bigcirc , F1T1–F1T9 at f = 1.0 Hz; \Box , F2T1–F2T9 at 1.0 Hz; \diamond , F3T1–F3F9 at 1.0 Hz; \triangle , F4T1–F4T9 at 1.0 Hz; \bigtriangledown , F1T1–F1T9 at 0.5 Hz; and \bullet , F1T1–F1T9 at 2.2 Hz.

Spagnolie et al. (2010); that is, the efficient wings or fins should flap on a time scale which is comparable to the relaxation time of the compliant structure (i.e., $\xi \simeq \pi/2$); otherwise, the compliance is either negligible (i.e., $\xi \ll \pi/2$) or dominant (i.e., $\xi \gg \pi/2$) in fluid–structure interaction and no propulsive performance improvement is expected in either case.

6. Further comments on the half- π phase delay condition

As we have found, the main strategy to enhance the thrust generation on a flexible pitching panel in a quiescent fluid is to have the maximum effective pitching angle (deformation) during the acceleration stage, closer to the instant of maximum angular velocity. As a result, (i) surrounding fluid is dragged more into the streamwise direction at the acceleration stage; (ii) entrained fluid is pushed more into the streamwise direction in the wake by the delayed shedding of TEV (i.e., weak interaction between the TEVs) near the instant of maximum angular velocity; and (iii) the induced fluid has a less detrimental influence at the deceleration stage. Therefore, the decay of jet-velocity after its peak becomes slower and the strong streamwise momentum is retained longer in the wake.

In previous sections, we have shown that the suggested mechanism is valid within the ranges of considered planform geometry, pitching frequency, and compliance. This is supported more clearly by collecting all the data measured in the present study. Fig. 16a shows the mean streamwise momentum flux coefficient ($\overline{C}_{M\lambda}$) through the horizontal plane of x/c = 2.5 ($-3.0 \le y/c \le 3.0$), calculated from the time-averaged velocity (\overline{n}) fields for all considered cases (see Eqs. (3) and (5) for the definitions). The position of calculating momentum flux was determined based on our analysis on the TEV circulation (Fig. 9), which has a dominant influence on the thrust generation. As shown, for each data set, the momentum flux coefficient increases as the phase difference approaches $\xi/2$ and the maximum value is achieved at $\xi \simeq 0.5\pi$ among the considered 29 cases. Although the momentum flux needs to be calculated at multiple planes located along the spanwise direction and summed up to fully estimate the thrust force (the scatters may come from this issue), the trend of optimal condition can be evidenced from the fact that the momentum flux at the mid-span plane has the dominant contribution to the thrust. Similar argument has been given by DeVoria and Ringuette (2012), as well. As shown in Fig. 16b, for each data set (with varying planform shape, pitching frequency and stiffness), the mean streamwise momentum coefficient measured at the center-plane shows a nice linear correlation with the measured mean thrust coefficient. This again shows

that the present explanation of the half- π phase delay condition with the flow fields at the mid-chord is valid.

Previously, several studies have suggested the phase difference between the pitching and bending motions as a possible parameter to determine the performance of the thrust generation irrespective of the presence of the freestream flow. Ahlborn et al. (1997) performed experiments with a two-dimensional foil which flaps from the rest and returns, and claimed that the thrust is enhanced by introducing a certain time delay prior to the return-flap. Heathcote et al. (2004) and Heathcote and Gursul (2007) suggested that the thrust performance of a flexible heaving foil without or with a freestream is a function of the Strouhal number and pitch phase angle defined as the phase angle of the displacement of trailing edge relative to the leading edge. With a free stream, the thrust was maximized at the pitch phase angles of 110°–120° and its efficiency was maximum at 95°–100° (Heathcote and Gursul, 2007). In Heathcote and Gursul (2007), relevant data for engineered and nature's thrust-generating undulating propulsors were also collected and their pitch phase angles for maximum efficiency covered a broad range of 75°-100°. On the other hand, Matsumoto et al. (2010) analyzed the flutter motion of two-dimensional plate and showed that the $\pi/2$ phase lag of pitching behind the heaving is an optimal value for the propulsion. For a self-propelled flapping flyers, Ramananarivo et al. (2011) has reported an optimal phase lag of $\pi/4$ for maximum aerodynamic performance, smaller than the expected optimal condition of $\pi/2$. Furthermore, for the cases with flexibility in multi-directions, i.e., isotropic flexibility, the performance of a flapping propulsion would be determined by the complex combinations of more parameters, as identified by Kang et al. (2011) who drew a nice scaling relation for the propulsion efficiency in terms of wing geometry, kinematics, structural properties, and fluid media. Since the range of optimal phase angle is guite broad depending on the actual operating conditions, this implies that the proposed half- π phase delay condition needs to be treated carefully before real applications. Recently, biorobotics research group in Seoul National University has materialized this half- π delay condition and demonstrated its usefulness empowered by a variable stiffness mechanism (Park et al., 2014).

7. Concluding remarks

In the present study, we have performed a series of particle image velocimetry measurements around pitching panels while varying its flexibility, planform shape and pitching frequency to explain the half- π phase delay condition in a quiescent fluid (Park et al., 2012) which states that the maximum thrust is generated at approximately $\pi/2$ phase lag of the bending angle behind the pitching angle. The phase delay of $\pi/2$ indicates that the panel has the maximum camber when its angular velocity is maximum, i.e., a larger effective pitching angle (deformation) is achieved during the accelerating stage. As a result, fluid is entrained more during the acceleration stage and is pushed more into the streamwise direction in the wake by the delayed shedding of TEV and fluid is dragged less to have a detrimental influence at deceleration stage. This was quantitatively confirmed by the calculation of the TEV circulation and momentum flux in streamwise direction as the phase lag approaches $\pi/2$, but is reduced significantly as the panel becomes stiffer (as the phase delay decreases). We also found that the present explanations are applicable to different panel geometry and pitching frequency under the zero-free-stream condition. In particular, we suggested that the over-compliance (with a phase delay larger than $\pi/2$) significantly weakens the TEV and the thrust generation, due to the increased energy waste in a fluid–structure interaction during the stroke reversal.

Considering the ranges of present parameters and the reports from previous studies, the present explanation of the maximum thrust-generation condition may need to be modified if there is a free-stream. With a free-stream, additional flow structures (e.g., leading-edge vortex) may exist in the wake and the propulsive performance is affected by positive or negative interactions between vortical structures. The convection velocity of the identified vortices into the wake would be affected as well. As explained, the optimal phase delay for the maximum thrust and efficiency is measured or found to be widely scattered around $\pi/2$ with a free-stream. Thus, further research is required to address the effect of free-stream on the validity of the explanation of half- π phase delay condition suggested in the present study.

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Appendix A. Supplementary data

Supplementary data associated with this article can be found in the online version at http://dx.doi.org/10.1016/j.jfluidstructs. 2016.10.004.

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