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Hydrodynamic advantages of a low aspect-ratio flapping foil

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ABSTRACT

A high aspect-ratio foil is known to be advantageous in terms of both thrust and efficiency in flapping propulsion. However, many species of fish have evolved a low aspect-ratio hydrofoil, which naturally leads one to search for its physical advantages in locomotion. Here we study the flow physics of a hydrofoil in angular reciprocating motion with negligible free-stream velocity to reveal the effects of an aspect ratio on hydrodynamic performance. By establishing a scaling law for the thrust of a foil of general shapes and corroborating it experimentally, we find that the thrust of an angularly reciprocating foil is maximized at a low aspect ratio of 0.7 while hydromechanical efficiency continuously increases with an aspect ratio. This result suggests that a low aspect-ratio foil can improve thrust produced by the foil when they start from rest, but at the expense of efficiency. © 2017 Elsevier Ltd. All rights reserved.

1. Introduction

Most of swimming and flying animals flap their fins or wings to propel through a surrounding fluid. The topics of flapping locomotion that have been addressed thus far include measuring the precise kinematics of animals (Fish and Lauder, 2006; Shelton et al., 2006), analyzing the flow structure and forces of a flapping foil (von Ellenrieder et al., 2003; Dong et al., 2006; Buchholz and Smits, 2008; Green and Smits, 2008; Kim and Gharib, 2013) and examining the role of an aspect ratio and flexibility of fish-like locomotion (Dewey et al., 2013; Raspa et al., 2014; Feilch and Lauder, 2015; Quinn et al., 2015; Yeh and Alexeev, 2016). One of the important issues in the mechanics of flapping propulsion is the effect of an aspect ratio of the foil. Early studies have proposed that a high aspect-ratio foil is advantageous in terms of both thrust coefficient and hydromechanical efficiency (Chopra, 1974; Chopra and Kambe, 1977; Cheng and Murillo, 1984; Karpouzian et al., 1990; Dong et al., 2006; Buchholz and Smits, 2008; Green and Smits, 2008; Dewey et al., 2013).

However, many biological studies revealed clear distinction of the caudal fin aspect ratio between migratory and nonmigratory fish (Nursall, 1958; Webb, 1984; Domenici and Blake, 1997; Flammang and Lauder, 2009; Domenici and Kapoor, 2010). The data on the aspect ratio are summarized in Fig. 1. The highly migratory fish, which are expected to give priority to efficiency improvement in cruising, have a high aspect-ratio caudal fin as shown in Fig. 1a. On the other hand, nonmigratory fish generally have a smaller aspect ratio than migratory fish. The comparison in normalized coordinates in Fig. 1c,d clearly shows the stark difference of the caudal fin shape between migratory and non-migratory fish. We further compared the distribution of the caudal fin aspect ratio of highly migratory and non-migratory marine fish in Fig. 1e (refer

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Experimental parameters.		
h	Height	1.5–22.5 cm
w	Width	1.2-20 cm
Λ	Sweepback angle	0–53 deg
ls	Side length	0–20 cm
$2\theta_m$	Stroke angle	35 deg
f	Frequency	0.1–2.5 Hz
R	Radius of rotation	13.2-32 cm
S	Surface area	$24-96 \text{ cm}^2$

Table 1

to Supplementary Material for the definition of an aspect ratio and the data of the aspect ratio of aquatic animals). In general, the former group has a caudal fin of a high aspect ratio, whereas the latter has a caudal fin of a much smaller aspect ratio.

The comparative studies on the fish caudal fins of diverse species found that a high aspect ratio is advantageous for efficiency improvement in cruising while a low aspect ratio is advantageous for thrust maximization in sprinting (Nursall, 1958; Webb, 1984; Domenici and Blake, 1997; Flammang and Lauder, 2009; Domenici and Kapoor, 2010), which is inconsistent with the conventional hydrodynamic argument that a high aspect ratio is advantageous in terms of both thrust and efficiency (Chopra, 1974; Chopra and Kambe, 1977; Cheng and Murillo, 1984; Karpouzian et al., 1990; Dong et al., 2006; Buchholz and Smits, 2008; Green and Smits, 2008; Dewey et al., 2013). Such controversy has not been treated or resolved by fluid-dynamic theory thus far. In this study, we investigate physical mechanisms underlying the advantages of a low aspect-ratio foil and resolve the inconsistent views on the role of an aspect ratio on thrust and efficiency.

In this study, we investigate the hydrodynamics of a flapping foil without steady forward motion as an elementary model of the situation where the caudal fin is used to propel from rest and especially focus on the effects of an aspect ratio on thrust and efficiency. Despite extensive fluid dynamic studies on flapping locomotion, most of them have considered heaving and/or pitching of a foil in steady forward motion or against incoming freestream. The hydrodynamics of a flapping foil without steady forward motion has drawn relatively less scientific interest. The vortex formation around a flapping foil that rotates from rest without forward motion has been investigated recently (Ahlborn et al., 1997; Kim and Gharib, 2011; DeVoria and Ringuette, 2012). The theoretical model for the thrust of an angularly reciprocating rectangular plate has been suggested based on flow visualization (Lee et al., 2013). However, the effect of an aspect ratio on thrust and efficiency in such a motion has not been addressed by the previous studies.

2. Experimental apparatus

An angularly reciprocating foil as an elementary model of a flapping foil at the start of locomotion is shown in Fig. 2. The foil was immersed in a transparent water tank of 75, 55, and 33 cm in the *x*-, *y*- and *z*-directions, respectively. The foil is in a single degree-of-freedom sinusoidal oscillation about the *z*-axis, and the rotating axis is fixed in space. We employed rectangular, trapezoidal and cropped delta foils of various dimensions; the ranges of design dimensions are height $h = [1.5 \ 22.5]$ cm, width $w = [1.2 \ 20]$ cm, sweepback angle of quarter-chord line $\Lambda = [0 \ 53]$ deg, and side length $l_s = [0 \ 20]$ cm. The foil is hape used in this study roughly mimics a caudal fin. The foil is attached to a metallic rod of length l = 12 cm which oscillates about the *z*-axis. The radius of rotation of the foil tip is $R = [13.2 \ 32]$ cm. While a rotating axis is on the chord of a foil in usual pitching and/or heaving models, in our model, a rotating axis is outside the chord of a foil. The stroke angle of the sinusoidal rotation $2\theta_m$ is fixed to 35° . The flapping frequency *f* ranges from 0.1 to 2.5 Hz. The experimental parameters are summarized in Table 1. The Reynolds number *Re* is defined as Re = Uw/v where *U* and *v* are characteristic velocity and kinematic viscosity, respectively. In our experiments, the Reynolds number Re, based on the period-averaged speed at 0.7 radius length $U = 4(0.7R)\theta_m f$ (Techet, 2008), ranges from 1.3×10^3 to 1.1×10^5 . The other dimensionless parameters are varied such that the aspect ratio $A = h^2/S = [0.075 \ 17]$ where *S* is a surface area, the normalized stroke amplitude $\beta = 2R\theta_m/w = [0.96.7]$, the height-to-width ratio $\eta = h/w = [0.075 \ 17]$, and the taper ratio $\lambda = l_s/w = [0 \ 1]$.

The velocity and vorticity fields were obtained from two-dimensional Digital Particle Image Velocimetry (DPIV) using hydrogen bubbles as seeding particles. The hydrogen bubbles are generated by the electrolysis of water using a platinum wire of 50 - m. The electrical power applied to the platinum wire is controlled by a power supply (Kenex HV-401) to maintain the size of hydrogen bubbles similar to the diameter of the platinum wire. To visualize the bubbles, the central plane perpendicular to the flapping tail was illuminated using a 2 W continuous laser with a 532 nm wavelength. The captured images were analyzed using DPIV software to generate velocity fields with an interrogation window size of 32×32 pixels with a 50% overlap. The processed velocity vectors were validated using a dynamic mean value operator. The number of error vectors was within 2% of the total number of velocity vectors. The thrust generated by the foil in the *y*-axis was also measured by a miniature load cell (Kyoto 333FB) at a sampling rate of 2 kHz. The capacity of a load cell is 4.9 N, and the precision is 0.1%, corresponding to the accuracy of about 0.005 N.



Fig. 1. (a) Highly migratory fish with a high aspect-ratio caudal fin. From left to right: tuna, swordfish and bonito. (b) Non-migratory fish with a low aspect-ratio caudal fin. From left to right: zebrafish, black stripe topminnow and rice fish. (c) The caudal fin profiles of highly migratory fish normalized by maximum length of the caudal fin: tuna (black), swordfish (red) and bonito (blue). (d) The caudal fin profiles of non-migratory fish normalized by maximum length of the caudal fin: zebrafish (black), stripe topminnow (red) and rice fish (blue). (e) Distribution of the aspect ratio of highly migratory marine fish (purple) and non-migratory marine fish (green). The aspect ratio *A* is defined as h^2/S where *h* is the maximum (tip to tip) span of the fin and *S* is the projected area of a fin planform. Also refer to Supplementary material. (For interpretation of the references to colour in this figure legend, the reader is referred to the web version of this article.)



Fig. 2. Schematic illustration of the experimental apparatus.



Fig. 3. (a) Thrust of a rectangular foil versus an aspect ratio. (b) Thrust below the slenderness limit versus $\rho h^2 w R \theta_m f^2$. A solid line is from the added-mass force theory. The black, red and blue symbols correspond to the frequency of f = 1, 1.5, 2 Hz, respectively. The squares and circles correspond to the rectangular foil with the area of 24 and 30 cm², respectively.

3. Results

3.1. Scaling law and added-mass theory

We first measured the thrust of a rectangular foil by varying an aspect ratio *A* over two orders of magnitude as shown in Fig. 3a. The trend in thrust curves changes at the aspect ratio of about 0.7. Before the transition, the thrust rapidly increases with the aspect ratio. However, after the transition, the thrust decreases gradually with the aspect ratio. For a slender foil with an extremely low aspect ratio, the hydrodynamic force can be modeled as an added-mass force based on the potential flow assumption (Lighthill, 1970; Ellington, 1984). The thrust is scaled as the product of the added mass $\sim \rho h^2 w$ and the acceleration $\sim R\theta_m f^2$:

$$T \sim \rho h^2 w R \theta_m f^2. \tag{1}$$

The thrust for an aspect ratio smaller than 0.7 is successfully predicted by (1), as shown in Fig. 3b, where thrust data measured experimentally collapse onto a straight line when plotted according to the scaling law (1). However, as the aspect ratio increases beyond the slenderness limit ($A \approx 0.7$), the added-mass theory based on the potential flow assumption is no longer valid since vortical structures generated by the foil should be taken into consideration.

3.2. Vortical structure of a flapping foil

In order to find the force production mechanism beyond the slenderness limit, we first identify the flow structures generated by a flapping foil as shown in Fig. 4a. The thrust of the angularly reciprocating foil is produced by the reaction of the momentum imparted to the vortical structures. When the foil accelerates to rotate, the vortical structures of an acceleration phase (APS: acceleration phase vortical structure) are formed at the edges of the foil (Fig. 4a(i-ii)). When the foil decelerates and reverses its direction, a strong vortical flow (SRV: stroke reversal vortical structure) is generated around the foil edges, as shown in Fig. 4a(iii–iv). The flapping foil, then, experiences thrust as shown in the sharp force peak around the stroke reversal of Fig. 4b, as reaction of the momentum imparted to the SRV. The SRV is shed away from the flapping foil immediately after the stroke reversal as shown in Fig. 4a(v). When the foil rotates after the stroke reversal, a second APV is generated by foil acceleration, as shown in Fig. 4a(v-vi). The momentum transfer through the generation of the APV is another source of thrust, as indicated by a second force peak in the time history of the thrust in Fig. 4b. The APV follows the motion of the foil until the next stroke reversal, and the vorticity separated at the foil edges continuously feed into the APV as shown in Fig. 4a(i–ii) and Fig. 4a(v–vi). Although not presented here, for all foil geometries considered in this study, SRV and APV were identified in flow visualization, and the peaks corresponding to SRV and APV were found in thrust curves.

We note that flapping frequency is different between Fig. 4a and b due to the limitation of our experimental apparatus. While the power of laser source limited PIV experiment to the flapping frequency below 0.2 Hz, force measurement for the flapping frequency below 0.5 Hz was not available due to basic noise level in our load cell. However, we suppose that this limitation in our experimental apparatus does not affect our arguments based on the observation that basic flow patterns –two distinct vortical structures per half cycle –does not change in a frequency range from 0.2 to 2 Hz.



Fig. 4. (a) Velocity vectors and vorticity contours around a trapezoidal foil with [h, w] = [4, 12] cm, oscillating at 0.2 Hz. (i)-(vi) correspond to dimensionless time tf = 0.04, 0.18, 0.29, 0.36, 0.43 and 0.46. Round arrows indicate the instantaneous rotating direction of the foil. Blue and red contours correspond to clockwise and counterclockwise vorticities, respectively. We denote a vortical structure in a counterclockwise stroke as APV, a vortical structure in a stroke reversal as SRV, and a vortical structure in a clockwise stroke as APV. (b) Thrust of a trapezoidal foil with [h, w] = [4, 12] cm, oscillating at 0.5 Hz. Black and red arrows indicate that the black and red curves correspond to stroke angle θ and thrust T, respectively. The times corresponding to (i)-(vi) of (a) are presented on the displacement curve of (b). (For interpretation of the references to colour in this figure legend, the reader is referred to the web version of this article.)

3.3. Scaling law and vortical structure

Based on the observed flow structures in Section 3.2, we construct a scaling law for the thrust of the foil of the aspect ratio larger than the slenderness limit ($A \approx 0.7$). The period-averaged thrust, T, is a function of fluid density and dynamic viscosity (ρ , μ), stroke frequency (f), stroke angle (θ_m), radius of rotation (R), foil dimensions (h, w), sweepback angle (Λ), and side length (l_s): $T = fn(\rho, \mu, R, h, w, f, \theta_m, \Lambda, l_s)$. Applying the Buckingham Pi theorem, one obtains the following dimensionless relationship:

$$\frac{T}{\rho w^4 f^2} = f(Re, A, \beta, \eta, \Lambda, \lambda), \tag{2}$$

where Re = Uw/v is the Reynolds number, $A = h^2/S$ is the aspect ratio, $\beta = 2R\theta_m/w$ is the normalized stroke amplitude, $\eta = h/w$ is the height-to-width ratio, and $\lambda = l_s/w$ is the taper ratio.

The thrust is produced by reaction of the momentum imparted to the vortical structures, i.e., APV and SRV. The periodaveraged thrust, *T*, can then be scaled as a product of the frequency, *f*, and the momentum imparted to the vortical structures per cycle, ΔI ; $T \sim f \Delta I$. The momentum imparted to the fluid consists of two parts associated with the APV and SRV as $\Delta I \sim I_A + I_S$, where subscripts *A* and *S* denote APV and SRV, respectively. Vorticity is concentrated in a separated shear layer around the foil edges. Thus, the momentum induced by the vortical structure is modeled as $I_i \sim \rho \Gamma_i S$, where Γ_i is the circulation of the vortical structure, *S* is the area of the foil, and the subscript *i* is either *A* or *S* (Wu et al., 2006). The circulation Γ_i is scaled with the strength of the vortex at the bottom tip. Therefore, the period-averaged thrust can be expressed as $T \sim (I_A + I_S)f \sim \rho(\Gamma_A + \Gamma_S)Sf$.

The scaling relation for Γ_i is obtained by applying the following physical constraint. The pressure difference across the front and back surfaces should be zero at the bottom tip of the foil where a vortex sheet is separated into the flow field (Theodorsen, 1935; Wu et al., 2006). Two distinct effects inducing the pressure difference at the tip should counterbalance each other: one is the acceleration of the foil, and another is the tangential velocity discontinuity induced by the vortex sheet (Theodorsen, 1935; Wu et al., 2006). This physical constraint allows us to write $\Delta p_a \sim \rho (\Delta U_t)^2$, where Δp_a is pressure difference induced by the acceleration and ΔU_t is tangential velocity difference between the front and back surfaces at the tip.

The acceleration of the foil *a* varies along the width-wise direction, whereas it is a constant along the height-wise direction. Therefore, the pressure difference at the tip induced by acceleration, Δp_a , can be scaled using the added mass

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force of the blade element with a width Δw and a height $h, F_a \sim \rho h^2 \Delta w a$. The pressure difference is then scaled as $\Delta p_a \sim F_a/(\Delta w h) \sim \rho h a$. Next, the tangential velocity discontinuity, ΔU_t , can be related to the strength of the vortex at the bottom tip as $\Delta U_t \sim \Gamma_i/w$ because the vortex sheet on the foil surface is separated tangentially and evolves into a tip vortex at the foil tip. Therefore, the characteristic circulation is expressed as $\Gamma_i \sim w h^{1/2} a_i^{1/2}$. Here the subscript *i* of acceleration *a* and circulation Γ is *A* for APV and *S* for SRV. Assuming that the motion of the foil follows a simple harmonic function in time, the deceleration and acceleration of the foil can be scaled as $a_A \sim a_S \sim R \theta_m f^2$, where $R \theta_m$ is the distance traveled by the foil tip. Then, the characteristic circulation can be also scaled as a single expression, $\Gamma_A \sim \Gamma_S \sim w h^{1/2} R_m^{1/2} \theta_m^{1/2} f$. Finally substituting this scaling for the circulation into the scaling law for the thrust $T \sim \rho(\Gamma_A + \Gamma_S)Sf$, the thrust is given by

$$T \sim \rho Sw h^{1/2} R^{1/2} \theta_m^{1/2} f^2.$$
(3)

In order to validate our scaling law, we collected thrust data of 195 cases by varying foil dimensions, flapping frequency, stroke amplitude, and foil shapes, and compared them with relation (3). As shown in Fig. 5b, all thrust data collapse onto a straight line when plotted according to (3). Meanwhile, the data are scattered when plotted versus $\rho SR^2 \theta_m^2 f^2$ in Fig. 5a, which corresponds to the conventional scaling of hydrodynamic force $\rho U^2 S$ with $U \sim R\theta_m f$. We further examine the dimensionless relation of the thrust by dividing both sides of (3) with $\rho w^4 f^2$:

$$\frac{T}{\rho w^4 f^2} \sim \frac{\eta^{5/2} \beta^{1/2}}{A}.$$
(4)

The dimensionless thrust is a function of the height-to-width ratio η , the normalized stroke amplitude β , and the aspect ratio A, but is independent of the Reynolds number *Re*, the sweepback angle Λ , and the taper ratio λ . In the range of *Re* studied in this work, our dimensionless scaling law is in good agreement with the experimental measurements, as shown in Fig. 5c.

3.4. Optimal aspect ratio

Based on the scaling law, we investigate the effects of the aspect ratio on thrust and efficiency. We note that (3) can be rearranged as

$$\tilde{T} = \frac{T}{\rho(R\theta_m)^{1/2} S^{7/4} f^2} \sim A^{-1/4}.$$
(5)

In other words, the scaled thrust, \tilde{T} , is dependent on only A for the given stroke amplitude $R\theta_m$, foil area S, and frequency f. The scaled values of experimentally measured thrust, \tilde{T} , is plotted versus A in Fig. 6a. The trend in the thrust curve is abruptly changed at the slenderness limit, the aspect ratio of approximately 0.7. Before the transition, the thrust rapidly increases with the aspect ratio A, as predicted by the scaling law (1). However, beyond the slenderness limit, the thrust decreases with the aspect ratio, $\tilde{T} \sim A^{-1/4}$, as predicted by the scaling law (5). Therefore, the aspect ratio around the slenderness limit, $A \approx 0.7$ is optimal in maximizing the thrust for the given foil area and kinematics.

We next examine the relationship between an aspect ratio and efficiency. In general, the hydromechanical efficiency of a flapping foil is defined as a ratio of the work used for propulsion to the total work conducted through foil motion, i.e., the Froude efficiency (Lighthill, 1970; Sfakiotakis et al., 1999). However, the efficiency of our foil model cannot be obtained using the conventional definition because the work used for propulsion is zero owing to the absence of forward motion. Instead, we introduce the hydrodynamic cost of endurance as a measure of efficiency, which is analogous to the aerodynamic cost of endurance in hovering insects (Wang, 2008; Lentink and Dickinson, 2009; Pesavento and Wang, 2009). The hydrodynamic cost of endurance *C* is defined as a dimensionless power to produce a unit thrust, $C = Wf/(TU_r)$, where *W* is the work conducted by the tail motion defined as $W = \int_0^{1/f} \tau \cdot \Omega dt$ with torque τ and angular velocity Ω . The reference velocity $U_r = \sqrt{2T/(\rho S)}$ is constant for a given tail area *S* and thrust *T*. The inverse of the hydrodynamic cost of endurance can be considered as the efficiency of the angularly reciprocating tail.

The scaling law for C can be obtained using the scaling relation for the thrust (3). For the work conducted by the foil motion: $W \sim TR\theta_m \sim \rho Swh^{1/2}R^{3/2}\theta_m^{3/2}f^2$, we get

$$C \sim \frac{R\theta_m}{w} A^{-1/4} \sim \beta A^{-1/4}.$$
(6)

The ratio of *C* to β is plotted versus the aspect ratio in Fig. 6b. The hydrodynamic cost of endurance decreases rapidly with the aspect ratio up to the slenderness limit, implying a drastic improvement in efficiency. The decreasing trend in the hydrodynamic cost of endurance with *A* weakens after the slenderness limit. C/β scales with $A^{-1/4}$ after the slenderness limit as predicted by the scaling law (6).

4. Discussion

From the experiment and modeling for thrust and efficiency, we are able to infer the following guidelines to determine the aspect ratio maximizing the propulsive performance at the start of locomotion. First, an aspect ratio below the slenderness



Fig. 5. Scaling relations of period-averaged thrust. (a, b) Plots of period-averaged thrust versus $\rho SR^2 \theta_m^2 f^2$ and the scaling law (3), respectively, for various foil dimensions and shapes. Black, red, and blue symbols correspond to the rectangular, trapezoidal, and cropped delta foils, respectively. Circles, triangles, and squares correspond to foils of w = 6, 9, and 12 cm, respectively. Empty, half-filled, and filled symbols correspond to foils of h = 4, 6, and 8 cm, respectively. (c) Dimensionless thrust, $T/(\rho w^4 f^2)$, versus $\eta^{5/2} \beta^{1/2} / A$ for various foil dimensions and shapes. Circles, triangles, and squares correspond to the cropped delta, trapezoidal, and rectangular foils, respectively. (For interpretation of the references to colour in this figure legend, the reader is referred to the web version of this article.)



Fig. 6. Effects of an aspect ratio, *A*, on thrust and efficiency. (a) Scaled thrust, $\tilde{T} = T/\rho(R\theta_m)^{1/2}S^{7/4}f^2$, of Eq. (5) versus an aspect ratio. (b) The ratio of hydrodynamic cost of endurance to normalized stroke amplitude, C/β , versus an aspect ratio. The vertical lines in (a) and (b) indicate the slenderness limit ($A \approx 0.7$). Squares, circles, triangles, right-pointing triangles, down-pointing triangles, left-pointing triangles, and diamonds correspond to the rectangular foils with the area of 24, 30, 36, 48, 54, 72 and 96 cm², respectively.

limit (≈ 0.7) should be avoided because it is unfavorable in terms of both thrust and efficiency. Second, for the purpose of thrust maximization, the aspect ratio around the slenderness limit, i.e., ≈ 0.7 , is preferred. On the other hand, the efficiency is continuously improved as the aspect ratio increases, although there is a loss in thrust above the slenderness limit. Maximizing the aspect ratio within the limitation of structural requirements is advantageous for the purpose of improving the efficiency. Therefore, the optimal aspect ratio of a propulsive system is determined by considering a tradeoff between thrust and efficiency whose relations to the aspect ratio are given in the scaling laws (5) and (6).

In order to examine whether our results favorably correspond to biological observations of actual fish, we further examine the caudal fin aspect ratio of non-migratory fish (data in Supplementary Material). The non-migratory fish is expected to have a higher priority on thrust maximization at the start of locomotion for immediate escape response from a predator. Only 2% of non-migratory fish are distributed within the aspect ratio below the slenderness limit (\approx 0.7), which is shown to be disadvantageous in terms of both hydrodynamic thrust and efficiency in our study. Majority of the non-migratory fish (\sim 51%) are distributed within the aspect ratio ranging from 0.7 to 1.5. While A = 0.7 corresponds to the slenderness limit that maximizes the thrust with given tail area and kinematics, at A = 1.5 the efficiency is expected to improve by about 17% despite a loss in thrust by 17% compared to the values at the slenderness limit. Furthermore, most of the non-migratory fish adopt following two strategies; 1) an aspect ratio below the slenderness limit is avoided, 2) a considerable number of non-migratory fish tune the aspect ratio of the caudal fin to be slightly above the slenderness limit. These results suggest that a large number of non-migratory fish species have selected a low-to-moderate aspect ratio fin by the tradeoff between thrust and efficiency at the start of locomotion.

5. Conclusion

In this study, by using scaling arguments for added-mass force and vortical impulse, we addressed a novel mechanism to explain the hydrodynamic advantage of a low aspect-ratio foil, which resolves the inconsistency of biological observations and conventional fluid dynamic viewpoints. The efficiency continuously improves as the aspect ratio increases, although there is a loss in thrust above the slenderness limit. On the other hand, for the purpose of thrust maximization, the aspect ratio around the slenderness limit, $A \approx 0.7$, is desirable. The aspect ratio below the slenderness limit is unfavorable in terms of both thrust and efficiency. For the start of locomotion, a low aspect-ratio foil near the slenderness limit can maximize thrust. Therefore, a low aspect-ratio foil, near the slenderness limit, can be a better choice than a high aspect-ratio foil for the situations where thrust generation for fast response from rest is crucial such as an escape from a predator (Domenici and Blake, 1997).

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Appendix A. Supplementary data

Supplementary material related to this article can be found online at http://dx.doi.org/10.1016/j.jfluidstructs.2017.03. 006.

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